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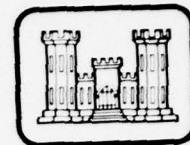




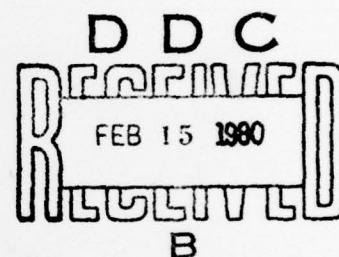
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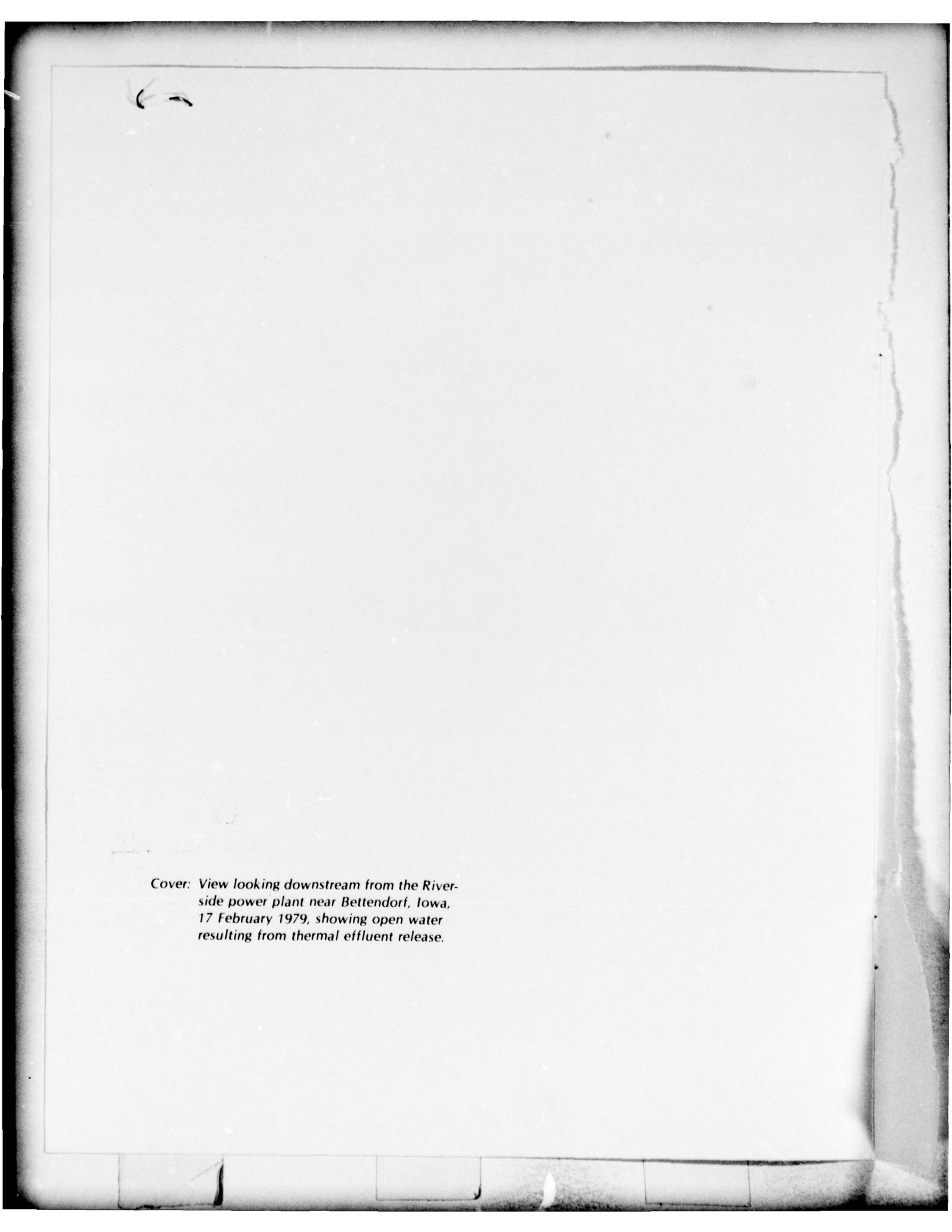
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*Cover: View looking downstream from the Riverside power plant near Bettendorf, Iowa, 17 February 1979, showing open water resulting from thermal effluent release.*



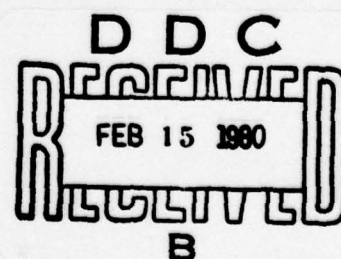
# CRREL Report 79-30



## *Suppression of river ice by thermal effluents*

George D. Ashton

December 1979



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## PREFACE

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# SUPPRESSION OF RIVER ICE BY THERMAL EFFLUENTS

George D. Ashton

## PART I. UNSTEADY SUPPRESSION OF RIVER ICE BY FULLY MIXED THERMAL EFFLUENTS

### Introduction

Imposition of a dam and a reservoir on a river generally results in the release of water during winter periods at a temperature above that which would ordinarily be experienced without the reservoir. As a result, the ice cover is suppressed in the reach downstream of the dam. The present analysis examines this ice suppression in an unsteady sense by including both the effects of unsteady variation of the meteorological variables and the storage and release of the energy associated with the melting and freezing of the ice cover. The effluent from the reservoir is assumed to be fully mixed over the initial cross section. In Part II of this report, the effects of a side discharge of thermal effluent will be considered so as to include the effects of lateral mixing.

Previous analytical work on the problem is sparse. Dingman et al. (1968) analyzed the steady-state case, that is, assumed constant meteorological conditions and chose as the criterion for the location of the ice edge downstream the point where the water temperature was 0°C. They well recognized the limitations of their steady-state model (Weeks and Dingman 1972). Paily et al. (1974) considered a similar problem but with a step increase in the effluent temperature and inclusion of the effect of longitudinal dispersion. Their criterion for ice edge location was also the location of the 0°C isotherm for open surface conditions. Examination of their results also shows that inclusion of the longitudinal dispersion term has small effect compared with simple nondispersive routing.

In contrast to these analytical studies Donchenko (1978) pointed out that the ice edge responded

strongly to changes in the meteorological variables and varying discharges, in one case fluctuating over a range of 80 km. Donchenko also pointed out the necessity of considering the hydrodynamic stability of the leading edge in locating the position of the ice front.

### Governing equations

For a flow with temperature uniformly mixed over the depth, with no transverse velocity variations, and steady flow, the governing partial differential equation is

$$\begin{aligned} \frac{\partial}{\partial t} (D\rho C_p T_w) + \frac{\partial}{\partial x} (DU\rho C_p T_w) &= \frac{\partial}{\partial x} \\ &\left( DE_x \frac{\partial (\rho C_p T_w)}{\partial x} \right) + \frac{\partial}{\partial z} \left( DE_z \frac{\partial (\rho C_p T_w)}{\partial z} \right) - \phi \end{aligned} \quad (1)$$

where  $D$  is the depth of flow,  $T_w$  is the water temperature,  $U$  is the mean velocity in the  $x$  (longitudinal) direction,  $\rho$  is the density,  $C_p$  is the specific heat capacity,  $z$  is the transverse coordinate, and  $E_x$  and  $E_z$  are respectively the longitudinal and transverse mixing coefficients.  $\phi$  is the heat flux at the top surface and there is assumed to be no flux at the bottom. If we assume  $D$ ,  $U$ ,  $\rho$  and  $C_p$  are constant, then the equation may be written in the form

$$\begin{aligned} \frac{\partial T_w}{\partial t} + U \frac{\partial T_w}{\partial x} &= \frac{\partial}{\partial x} \left( E_x \frac{\partial T_w}{\partial x} \right) \\ &+ \frac{\partial}{\partial z} \left( E_z \frac{\partial T_w}{\partial z} \right) - \frac{\phi}{\rho C_p D} \end{aligned} \quad (2)$$



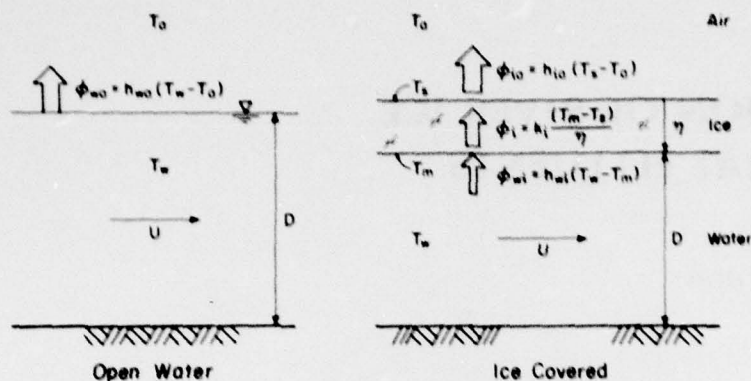


Figure 1. Definition sketch for heat transfer analysis.

We now neglect longitudinal mixing and obtain

$$\frac{\partial T_w}{\partial t} + U \frac{\partial T_w}{\partial x} = \frac{\partial}{\partial z} \left( E_z \frac{\partial T_w}{\partial z} \right) - \frac{\phi}{\rho C_p D} \quad (3)$$

If we further assume that  $T_w$  is fully mixed across the width, then

$$\frac{\partial T_w}{\partial t} + U \frac{\partial T_w}{\partial x} = - \frac{\phi}{\rho C_p D} \quad (4)$$

Finally, if a Lagrangian viewpoint is adopted we may write

$$\frac{DT_w}{Dt} = - \frac{\phi}{\rho C_p D} \quad (5)$$

#### Outline of analysis

A definition sketch is presented in Figure 1.

When the top surface of the flow is open to the atmosphere,  $\phi = \phi_{wa}$  where  $\phi_{wa}$  is the heat flux from the water surface to the atmosphere above.  $\phi_{wa}$  is a complicated function which has a number of components depending upon air temperature, wind velocity, humidity, cloud cover, time of day and other factors. For present purposes, we will assume the wind speed  $V_w$  and air temperature  $T_a$  are dominating and express  $\phi_{wa}$  by the simple relationship

$$\phi_{wa} = h_{wa} (T_w - T_a) \quad (6)$$

where  $h_{wa}$  is a heat transfer coefficient that is a function of the factors noted above. As a practical matter we will use, in the numerical simulation which follows, a relationship of the form

$$h_{wa} = a + b V_w \quad (7)$$

where  $a$  has dimensions of  $W m^{-2} ^\circ C^{-1}$  and  $b$  has dimensions of  $W m^{-3/2} s ^\circ C^{-1}$ . Depending upon the availability of input data, one may always calculate an equivalent value of  $a$ ,  $b$  or  $h_{wa}$  by energy budget analyses. In general,  $a$ ,  $b$  and  $h_{wa}$  are functions of time only, since the lengths over which the analysis is carried out are generally much shorter than corresponding spatial variations of the energy budget variables. However, our main purpose here is not to examine refinements in energy budget calculations but rather to examine the implications of the neglect of the unsteadiness of the total flux  $\phi$  and the energy required to melt (or freeze) an ice cover.

When the flow is ice-covered,  $\phi = \phi_{wi}$ , but now the heat flux from the water to the ice depends upon the flow variables. In particular, we take the heat flux from the water to the ice,  $\phi_{wi}$ , as

$$\phi_{wi} = h_{wi} (T_w - T_m) \quad (8)$$

where  $h_{wi}$  is a heat transfer coefficient and  $T_m$  is the melting point ( $T_m = 0^\circ C$ ). Closed conduit turbulent heat transfer correlations are generally of the form

$$\frac{h_{wi} R}{k_w} = C \left( \frac{U_w R \rho_w}{\mu} \right)^{0.8} \left( \frac{\mu C_p}{k_w} \right)^{0.4} \quad (9)$$

where  $R$  is the hydraulic radius (m),  $U_w$  is the average flow velocity ( $m s^{-1}$ ),  $\rho_w$  is the water density ( $kg m^{-3}$ ),  $\mu$  is the dynamic viscosity ( $kg m^{-1} s^{-1}$ ),  $C_p$  is the specific heat ( $J kg^{-1} ^\circ C^{-1}$ ), and  $k_w$  is the thermal conductivity of the water ( $W m^{-1} ^\circ C^{-1}$ ).  $C$  is an empirical coefficient on the order of 0.023 (see, e.g., Rohsenow and Choi 1961) for smooth surfaces. When the water is above

freezing and the flow is turbulent, relief features (ice ripples) form on the underside of the ice cover which increase the value of  $C$  by up to 50% (Ashton and Kennedy 1972). Evaluating the properties at  $0^\circ\text{C}$  and taking  $R = D/2$ ,

$$h_{wi} = C_{wi} \frac{U^{0.8}}{D^{0.2}} \quad (10)$$

where  $C_{wi} = 1622 \text{ W s}^{0.8} \text{ m}^{-2.6} ^\circ\text{C}^{-1}$  for  $C = 0.023$  and  $C_{wi} = 2433 \text{ W s}^{0.8} \text{ m}^{-2.6} ^\circ\text{C}^{-1}$  for  $C = 0.0345$ .

Equation 5 may now be integrated to yield, in co-ordinates moving with the flow,

$$\frac{T_w - T_a}{T_{w,0} - T_a} = \exp \left[ \frac{-h_{wa}(t-t_0)}{\rho C_p D} \right] \text{ open surface} \quad (11)$$

$$\frac{T_w - T_m}{T_{w,0} - T_m} = \exp \left[ \frac{-h_{wi}(t-t_0)}{\rho C_p D} \right] \text{ ice covered} \quad (12)$$

where the initial condition is taken as  $T_w = T_{w,0}$  at  $t = t_0$  and  $x = x_0$ . Relative to coordinates fixed in space such that  $dx = U dt$  the corresponding equations are

$$\frac{T_w - T_a}{T_{w,0} - T_a} = \exp \left[ \frac{-h_{wa}(x-x_0)}{\rho C_p U D} \right] \text{ open surface} \quad (13)$$

$$\frac{T_w - T_m}{T_{w,0} - T_m} = \exp \left[ \frac{-h_{wi}(x-x_0)}{\rho C_p U D} \right] \text{ ice covered.} \quad (14)$$

Equation 14 is shown in Figure 2 for typical parameter values.

The melting and thickening of the ice cover is governed by the energy balance at the water/ice interface

$$\phi_i - \phi_{wi} = \rho_i \lambda \frac{d\eta}{dt} \quad (15)$$

where  $\phi_i$  is the heat flux by conduction through the ice cover ( $\text{W m}^{-2}$ ),  $\rho_i$  is the density of ice ( $\text{kg m}^{-3}$ ),  $\lambda$  is the heat of fusion ( $\text{J kg}^{-1}$ ),  $\eta$  is the ice thickness (m), and  $d\eta/dt$  is the rate of thickening. The conductive flux  $\phi_i$  through the ice cover is treated in a quasi-steady manner by assuming a linear temperature profile through the ice thickness. Thus,

$$\phi_i = \frac{k_i (T_m - T_s)}{\eta} \quad (16)$$

where  $k_i$  is the thermal conductivity of the ice ( $\text{W m}^{-1}$

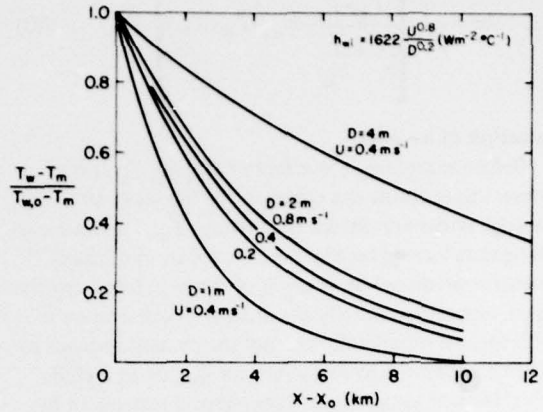


Figure 2. Downstream attenuation of water temperature beneath an ice cover.

$^\circ\text{C}^{-1}$ ),  $T_s$  is the top surface temperature of the ice cover, and we require that  $T_s \leq 0^\circ\text{C}$  because of the state relationship. In the absence of evaporation or condensation on the top surface,  $\phi_i = \phi_{ia}$  where  $\phi_{ia}$  is the heat flux from the surface to the atmosphere.  $\phi_{ia}$  may be calculated in a manner similar to  $\phi_{wa}$  through introduction of a heat transfer coefficient  $h_{ia}$  applied to the difference  $T_s - T_a$  in the form

$$\phi_{ia} = h_{ia} (T_s - T_a). \quad (17)$$

Using  $\phi_i = \phi_{ia}$  allows us to eliminate  $T_s$  between eq 16 and 17 and results in

$$\phi_i = \frac{T_m - T_a}{\frac{\eta}{k_i} + \frac{1}{h_{ia}}}. \quad (18)$$

We could also in a similar manner add the effects of a snow cover by introducing an additional term  $\eta_s/k_s$  in the denominator of eq 18.

Substitution of eq 18 in eq 15 then results in

$$\frac{T_m - T_a}{\frac{\eta}{k_i} + \frac{1}{h_{ia}}} - h_{wi}(T_w - T_m) = \rho_i \lambda \frac{d\eta}{dt} \quad (19)$$

which, if  $T_a$  and  $T_w$  are constant in time, may be readily integrated. They seldom are, however, so in the numerical analysis eq 19 will be integrated numerically in step-wise fashion in the form

$$\Delta\eta = \frac{1}{\rho_i \lambda} \left[ \frac{T_m - T_a}{\eta + \frac{1}{h_{ia}}} - h_{wi} (T_w - T_m) \right] \Delta t. \quad (20)$$

#### Location of ice edge

Before examining the effect of varying  $T_a$ , it is of interest to examine the criterion for the location of the ice edge under conditions of constant  $T_a$ . Previous investigators have generally used the location of the  $0^\circ\text{C}$  isotherm predicted by analysis of the heat loss from the open surface (essentially the analysis leading to eq 13) with various modifications from the present analysis to consider different formulations for  $\phi_{wa}$  or to include the effects of longitudinal dispersion. From eq 13 by the present analysis,

$$(x-x_0) \Big|_{T_w = 0^\circ\text{C}} = \frac{-\rho C_p U D}{h_{wa}} \left[ \log_e \left( \frac{-T_a}{T_{w,0} - T_a} \right) \right] \quad (21)$$

Since the heat balance at the surface determines whether or not ice will form at that surface, it is a better indication of the location of the ice edge than is the location of the  $0^\circ\text{C}$  isotherm. The condition is obtained by assuming incipient ice formation corresponds to  $d\eta/dt = 0$  when  $\eta = 0$ . From eq 19 this is at a temperature  $T_{w,e}$  given by

$$(T_{w,e} - T_m) = \frac{h_{ia}}{h_{wi}} (T_m - T_a). \quad (22)$$

Substituting into eq 13 the corresponding location is given by

$$(x-x_0) \Big|_{T_{w,e}} = \frac{-\rho C_p U D}{h_{wa}} \log_e \left[ \frac{T_m - T_a}{T_{w,0} - T_a} \left( 1 + \frac{h_{ia}}{h_{wi}} \right) \right] \quad (23)$$

It is now necessary to assume that  $h_{ia} = h_{wa}$ , which is reasonable at the ice edge since the top surface of the ice edge may be considered wet. The ratio of the distances by the two criteria is then given by

$$\frac{(x-x_0) \Big|_{T_{w,e}}}{(x-x_0) \Big|_{T_w = 0}} = \frac{\log_e \left[ \frac{T_m - T_a}{T_{w,0} - T_a} \left( 1 + \frac{h_{wa}}{h_{wi}} \right) \right]}{\log_e \left[ \frac{T_m - T_a}{T_{w,0} - T_a} \right]} \quad (24)$$

and, of course, the ratio is always less than unity for

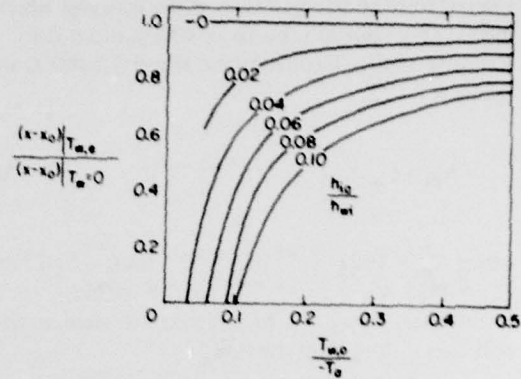


Figure 3. Comparison of distances downstream to edge of ice cover for the criteria of equilibrium heat fluxes and  $0^\circ\text{C}$  isotherm location, as a function of the water temperature/air temperature ratio.

$T_a < 0^\circ\text{C}$ . The ratio is presented in Figure 3 as a function of  $T_{w,0}/(-T_a)$  for various values of  $h_{ia}/h_{wi}$ . The ratio is small corresponding to small values of  $T_{w,0}/(-T_a)$  (either large initial water temperatures or small sub-freezing air temperatures) and large values of  $h_{ia}/h_{wi}$  (large wind speeds or slow flow velocities). Negative values of the ratio correspond, of course, to cases where  $\phi_{wi} < \phi_{wa}$ , which physically may be interpreted as those conditions for which ice will thicken because the heat loss to the atmosphere is greater than the heat delivered to the undersurface by the flow. To put the situation into better perspective, Figure 4 presents values of the ratio for particular values of  $T_{w,0}$ ,  $h_{wa}$ ,  $h_{wi}$  and  $T_a$ .

Which of the two criteria for the ice edge location is operative depends on the sequence of air temperatures. Clearly, water will not freeze until its temperature has decreased to  $0^\circ\text{C}$ . Thus, unless a prior ice cover is present, the most upstream location of the ice edge is given by the  $0^\circ\text{C}$  isotherm, corresponding to the criterion represented by eq 21. On the other hand, if an ice cover is present and the ice edge is receding downstream due to melting, it will only recede to the location at which the melting from below equals the tendency to thicken from above. Thus, during periods of warming air temperatures when the ice edge is receding downstream, the criterion of heat balance represented by eq 23 applies. During periods of cooling air temperatures, the  $0^\circ\text{C}$  isotherm location governs first ice appearance and eq 21 applies. In the simulation that follows, the order of logic in the calculation of ice thickening results in either the  $0^\circ\text{C}$  criterion or the heat balance criterion being naturally satisfied, although the latter is only approached asymptotically in time because of the unsteady nature of the calculations.



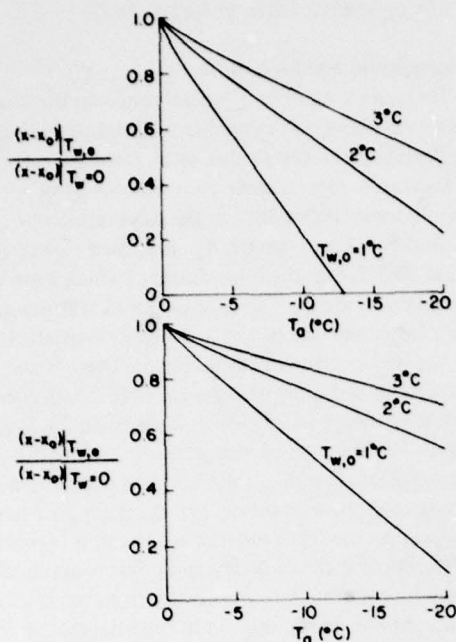


Figure 4. Comparison of distances downstream to edge of ice cover for the criteria of equilibrium heat fluxes and  $0^\circ\text{C}$  isotherm location as function of the air temperature and initial water temperature.

Figure 3 serves mainly to indicate the conditions for which the  $0^\circ\text{C}$  isotherm criterion is a poor approximation for the location of the ice edge under steady state conditions. The numerical simulation presented below is sufficiently simple computationally that the unsteady effects of  $T_a$  may also be included. The regions for which the ratio is negative, of course, are where the analysis presented herein is most useful, as the  $0^\circ\text{C}$  criterion would predict finite lengths of open water while, in fact, no open water will exist except in the very near field. This region also corresponds to the conditions which Silberman (1974), in a discussion of Paily et al.'s (1974) analysis, suggested resulted in plunging of the warm discharge beneath a cooler, but lighter, layer of near  $0^\circ\text{C}$  water. Such a plunging phenomenon undoubtedly occurs for small velocities and small densimetric Froude numbers and clearly is the case for warm water flowing into large bodies of water. Similarly, the reappearance of open water downstream from an ice-covered reach may be explained by the existence of a larger velocity downstream from a region of smaller velocity, since  $\phi_{wi}$  is increased more or less proportionally to the velocity. The effect would be more likely to occur where the lower velocity results from increased depth rather than from increased width,

since increased depth results in slower attenuation of  $T_w$  due to  $\phi$  (through the velocity effect) than occurs by simple reduction of velocity as a result of width increase (since  $U$  has small effect on the attenuation of  $T_w$ ). The above reasoning is perhaps made more clear by examination of the decrease in  $T_w$  with distance described by eq 14, which is presented in Figure 2. Again, the analysis about to be described would allow quantitative examination of specific cases.

#### Numerical simulation

The numerical simulation is quite straightforward. The temperature evolution along a reach is computed using eq 5 in the difference form

$$T_{wi}^{i+1} = T_{wi}^i - \frac{\phi}{\rho C_p D} \Delta t \quad (25)$$

where the superscript  $i$  denotes a time step, and the subscript  $j$  denotes a distance node. Since eq 25 is in the Lagrangian sense, this requires that  $\Delta x_j = U_j \Delta t$ . This requires, in turn, that the total reach be subdivided into subreaches with lengths varying if  $U$  varies but this presents no serious problem. Typical time steps used were of the order of 1200 to 1800 seconds. At the conclusion of each time step, the thickness of the ice was determined at each  $j$  node point using eq 20 written in the form

$$\eta_j^{i+1} = \eta_j^i + \frac{1}{\rho_i \lambda} \left[ \frac{T_m - T_a}{\frac{\eta_j^i}{k_i} + \frac{1}{h_{ia}}} - h_{wi} (T_{wi}^{i+1} - T_m) \right] \quad (26)$$

The initial temperature at the upstream end of the subreach was specified for each time as was the air temperature. For the examples presented, daily averages were used although it would be quite easy to specify a more detailed time variation. In the calculation of  $\phi$  at each  $j$  node point, it was necessary to use either eq 6 or eq 8, depending on whether the surface was open or ice-covered. In the thickness calculations,  $\phi_i$  was taken equal to zero if  $T_a > 0^\circ\text{C}$ . After the new ice thickness was calculated, it was set equal to zero if a negative value resulted. In the example simulations presented here,  $h_{wa}$  and  $h_{ia}$  were assumed equal.

The problem also requires specifications of initial conditions. In Figure 5 are presented two examples of the effect of specifying a constant-thickness initial ice cover of either zero or finite thickness.  $T_a$  was maintained at a constant value of  $-5^\circ\text{C}$ . The distance to ice edge stabilizes, after many days, for the finite initial thickness of 0.3 m and, after several days, for the zero initial thickness. In the latter case, the time

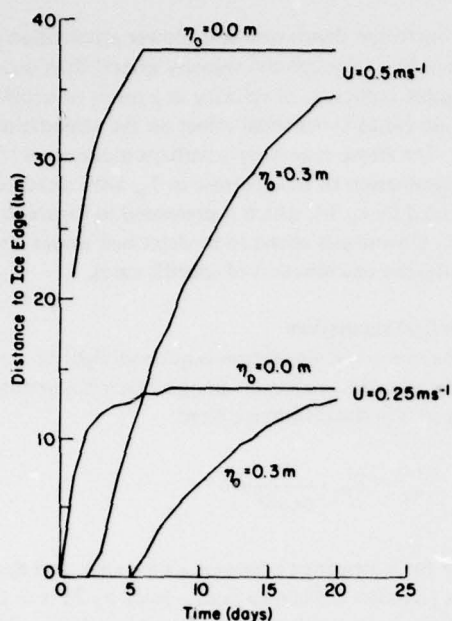


Figure 5. Example simulation of the movement of the ice edge in response to an abrupt release of thermal effluent for different assumed initial ice conditions.

to stabilize is longer than the time of travel to the equilibrium ice edge location because ice is grown until the temperature front reaches the ice edge and this ice, in turn, must be melted before equilibrium is achieved. A better set of initial conditions could probably be obtained by running the simulation for a few days prior to the period under investigation. Figure 5 also illustrates the potential errors in neglecting the energy required to melt the ice cover in the steady-state analyses referred to earlier.

There is one particular circumstance where the above analysis must be modified. When the air temperature is cooling, the ice edge location moves upstream. However, eq 26 is not applicable here until an ice cover exists and ice will not form until the water has cooled to  $0^{\circ}\text{C}$ . In this case, the  $0^{\circ}\text{C}$  isotherm location is the more correct location than the location given by the case corresponding to  $\phi_{wi} = \phi_{wa}$  as  $\eta \rightarrow 0$  (eq 23 in the steady-state case). There are thus two locations for the ice edge, depending on the prior history of the ice cover extent.

Finally, while no complete field data are available at the time of writing the simulation has been applied to the 1965 data presented by Dingman et al. (1967) but with an estimate of  $T_{w,0}$ ,  $U$  and  $D$ . The results are presented in Figure 6 and show reasonable agreement with the three observations available.

The program is listed in Appendix A.

#### Uncertainties and limitations

There are a number of uncertainties in the analysis presented above and a number of limitations to its applicability. Uncertainties, aside from the difficulties of accurately representing the meteorological variables, include some uncertainty in the calculation of  $h_{wa}$ ,  $h_{wi}$  and  $h_{ia}$ . For example,  $h_{wi}$  is known to increase by about 50% due to the relief features which form on the underside of the ice. Since these relief features appear as a consequence of  $\phi_{wi}$ , it is probably reasonable to use the higher value in calculations. There is also considerable uncertainty that the ice edge location predicted by use of eq 19, which leads to eq 23, is appropriate. For the case of increasing air temperatures and a downstream receding ice front, it is probably a good approximation. For the case of decreasing air temperatures and an ice front moving upstream, it is probably poorer because the hydrodynamic forces acting on a newly formed thin ice cover result in initial thickening by accumulation rather than by thermal thickening. One possible improvement would be to calculate the ice production under such cases and locate the ice front by use of the equilibrium accumulation thickness predicted by the analysis of Pariset and Hausser (1961). Even that approach, however, would be limited by the fact that above a velocity of about  $0.6 \text{ m s}^{-1}$  it is difficult for a cover to even form by such accumulation.

The limitations imposed by the simplifications of the analysis are several. Longitudinal mixing has been neglected but this is considered to have a negligible effect compared with the thermal inertia of the ice cover itself. The assumption of complete vertical mixing makes the analysis inappropriate for locations very close to the source of thermal effluent, unless the effluent is already fully mixed as is the case for reservoir discharges. The assumption of complete mixing across the width of the flow, and consequent neglect of lateral mixing and dispersion, is important both near the source of the effluent and well downstream near the location of the ice front. The source problem is reasonably self-evident. At the ice front, the limitation occurs because of variations in depth and velocity across the width of the flow which, because of the resulting lateral variations in  $dT_w/dt$ , cause a laterally uniform temperature distribution to become non-uniform. Thus, it is common that the ice front is observed to be somewhat V-shaped, with the apex of the V downstream and near the thalweg of the channel. These latter effects are currently under analysis by introducing lateral mixing into the formulation and require the additional consideration that the dispersion coefficient for closed surface flow is approximately half that for open surface flow at the same depth.

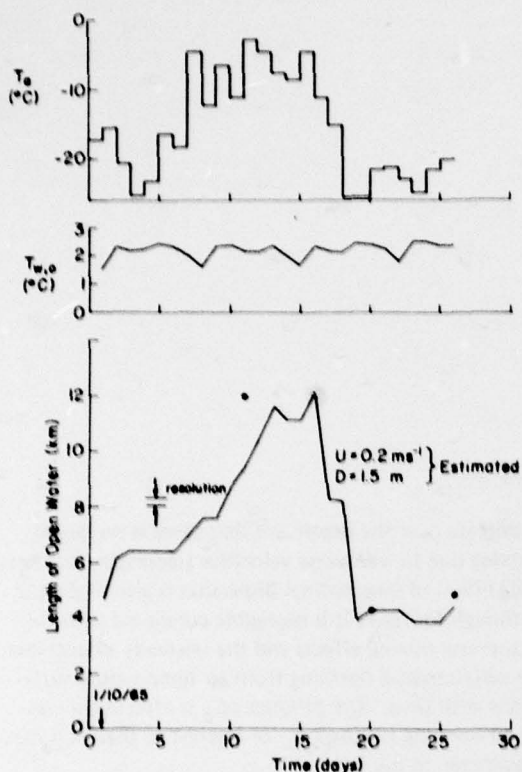


Figure 6. Comparison of simulation results to observations for Riverside location in 1965. Top figures are the air temperature and release water temperature variations. Bottom figure is the simulation result (solid line) and observations (plotted points).

Finally, the hydraulics of the flow have been assumed to be steady. This is a serious limitation but could, in principle, be overcome by frequent use of numerical models of open-channel flow to calculate the applicable velocities and depths at various stages of the simulation. These models should include the effect of ice cover on the flow but are substantially uncoupled from the thermal analysis.

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## PART II. EFFECT OF TRANSVERSE MIXING ON ICE SUPPRESSION

### Introduction

Rivers are commonly used for the disposal of thermal wastes and, in most cases, the disposal occurs in the form of a side channel discharge of heated effluent. During periods of ice cover, the effect of such effluents is to suppress the ice cover from its otherwise natural thickening and, in the vicinity of the release, open water often results. The intent of this work is to explore the extent of this ice suppression as a function of the parameters which have most significant effect on the suppression. Part I of this report dealt with the ice suppression resulting downstream from an effluent fully mixed with the flow as would be experienced downstream from a reservoir release or from a power plant with an outfall diffuser operated so as to provide complete mixing.

### Analysis of dispersion and heat loss

The governing differential equation is

$$\frac{\partial(\rho C_p T_w)}{\partial t} + U \frac{\partial(\rho C_p T_w)}{\partial x} = \frac{\partial}{\partial z} \left( E_z \frac{\partial(\rho C_p T_w)}{\partial z} \right) - \frac{\phi}{D} \quad (1)$$

where  $\rho$  is the density ( $\text{kg m}^{-3}$ ),  $C_p$  is the specific heat ( $\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ ),  $T_w$  is the water temperature ( $^\circ\text{C}$ ),  $E_z$  is a transverse dispersion coefficient ( $\text{m}^2 \text{ s}^{-1}$ ),  $\phi$  is the heat flux at the top surface ( $\text{W m}^{-2}$ ),  $D$  is the depth (m),  $x$  is the longitudinal coordinate,  $z$  is the transverse coordinate,  $U$  is the mean velocity, and  $t$  is time. Equation 1 implies that mixing of the effluent with the flow is

complete over the depth and that there is no lateral mixing due to transverse velocities (secondary currents). The effect of longitudinal dispersion is also neglected, although for rivers it is negligible compared with the transverse mixing effects and the unsteady effects due to variation in  $\phi$  resulting from air temperature variations with time. The product  $\rho C_p$  is effectively constant over the range of  $T_w$  of interest so that eq 1 may be written in the form

$$\frac{\partial T_w}{\partial t} + U \frac{\partial T_w}{\partial x} = \frac{\partial}{\partial z} \left( E_z \frac{\partial T_w}{\partial z} \right) - \frac{\phi}{\rho C_p D} \quad (2)$$

The heat flux  $\phi$  at the surface depends on whether or not an ice cover is present. If an ice cover is present,  $\phi$  may be reasonably represented by an expression of the form

$$\phi_{wi} = h_{wi} (T_w - T_m) \quad (3)$$

where  $\phi_{wi}$  is the heat flux from the water to the ice,  $h_{wi}$  is a heat transfer coefficient applied to the temperature difference  $T_w - T_m$  between the flow and the ice/water surface which is at a temperature  $T_m = 0^\circ\text{C}$  because of the state relation. The heat transfer coefficient  $h_{wi}$  depends on the flow variables. By analogy with closed conduit turbulent heat transfer,  $h_{wi}$  is determined from a Nusselt-Reynolds-Prandtl number correlation of the form

$$\frac{h_{wi} R}{k_w} = C \left( \frac{UR\rho}{\mu} \right)^{0.8} \left( \frac{\mu C_p}{k_w} \right)^{0.4} \quad (4)$$

where  $h_{wi} R/k_w$  is the Nusselt number,  $UR\rho/\mu$  is the

Reynolds number,  $\mu C_p / k_w$  is the Prandtl number,  $R$  is the hydraulic radius,  $k_w$  is the water thermal conductivity ( $\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$ ), and  $\mu$  is the dynamic viscosity ( $\text{kg m}^{-1} \text{ s}^{-1}$ ).  $C$  is somewhat uncertain but is of the order of 0.023 to 0.030 depending upon the under-surface roughness of the ice. The Prandtl number for water is 13.6 at  $0^\circ\text{C}$  and decreases with increasing temperature. At  $0^\circ\text{C}$ ,  $dPr/dT_w \approx -0.4 \text{ } ^\circ\text{C}^{-1}$ , so in the range  $0^\circ\text{C}$  to  $4^\circ\text{C}$  the error in assuming  $Pr$  constant yields at most about 5% error in calculation of  $h_{wi}$  and will be neglected in view of the uncertainty in  $C$ .

If the water surface is open to the atmosphere, then  $\phi$  depends on the energy budget at the water surface. For present purposes, we will approximate  $\phi$  by

$$\phi_{wa} = h_{wa} (T_w - T_a) \quad (5)$$

where  $\phi_{wa}$  is the heat flux from the water to the air,  $h_{wa}$  is a heat transfer coefficient applied to the difference between the water temperature  $T_w$  and the ambient air temperature  $T_a$ . Coefficient  $h_{wa}$  has large dependence on the wind velocity, generally of the form

$$h_{wa} = a + b V_w \quad (6)$$

where  $a$  is the heat transfer coefficient for still air and  $b V_w$  is the wind effect with  $V_w$  the ambient wind velocity at some specified distance above the surface. In numerical examples presented later,  $h_{wa}$  will be taken constant although the effect of eq 6 may easily be included in the numerical analysis (or, for that matter,  $h_{wa}$  can be calculated from even more detailed energy budget calculations of  $\phi_{wa}$ ).

The transverse mixing coefficient also depends upon whether or not the flow is ice-covered or open. We here relate  $E_z$  to a coefficient  $k$  times the product of the shear velocity  $U_*$  and hydraulic radius  $R$  in the form

$$E_z = k U_* R \quad (7)$$

where for open channel flow  $R \approx D$  and for covered flow  $R \approx D/2$ . Denoting  $E_z$  for open flow by  $E_{zo}$  and for covered flow by  $E_{zi}$ , then

$$E_{zo} = 2E_{zi} = k U_* D. \quad (8)$$

Engmann (1977) has measured  $E_z$  for both covered and open flows and found results consistent with eq 8, with  $k$  in the range 0.15 to 0.2 which may be compared to a commonly used value of 0.23 (Okoye 1970). However, if the river is not straight, considerably larger values of  $k$  are observed (Paily and Sayre 1978).

The situation for a partially open ice cover where the edge of the ice cover is more or less aligned with

the flow velocity is unclear. In open water areas far from the ice edge, it is reasonable to expect  $E_z = E_{zo}$ , while under the ice cover far from the edge  $E_z = E_{zi}$ . The nature of the numerical simulation presented below makes it convenient to assume an abrupt change in  $E_z$  at the ice edge. The finite difference algorithm used effectively causes a transition from  $E_{zo}$  to  $E_{zi}$  over a distance depending on the width of the grid elements.

#### Analysis of ice thickening and melting

The melting and thickening of the ice cover are governed by the energy balance at the water/ice interface (Fig. 1)

$$\phi_i - \phi_{wi} = \rho_i \lambda \frac{d\eta}{dt} \quad (9)$$

where  $\phi_i$  is the heat flux by conduction through the ice cover ( $\text{W m}^{-2}$ ),  $\rho_i$  is the density of ice ( $\text{kg m}^{-3}$ ),  $\lambda$  is the heat of fusion ( $\text{J kg}^{-1}$ ),  $\eta$  is the ice thickness (m), and  $d\eta/dt$  is the rate of thickening. The conductive flux  $\phi_i$  through the ice cover is treated in a quasi-steady manner by assuming a linear temperature profile through the ice thickness; thus,

$$\phi_i = \frac{k_i (T_m - T_s)}{\eta} \quad (10)$$

where  $k_i$  is the thermal conductivity of the ice ( $\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$ ), and  $T_s$  is the top surface temperature of the ice cover and we require that  $T_s \leq 0^\circ\text{C}$  because of the state relationship. In the absence of evaporation or condensation on the top surface,  $\phi_i = \phi_{ia}$ , where  $\phi_{ia}$  is the heat flux from the surface to the atmosphere.  $\phi_{ia}$  may be calculated in a manner similar to  $\phi_{wa}$  through introduction of a heat transfer coefficient  $h_{ia}$  applied to the difference  $T_s - T_a$  in the form

$$\phi_{ia} = h_{ia} (T_s - T_a). \quad (11)$$

Using  $\phi_i = \phi_{ia}$  allows us to eliminate  $T_s$  between eq 10 and 11 and results in

$$\phi_i = \frac{T_m - T_a}{\frac{\eta}{k_i} + \frac{1}{h_{ia}}} \quad (12)$$

We could also in a similar manner add the effects of a snow cover by introducing an additional term  $\eta_s/k_s$  in the denominator of eq 12.

Substitution of eq 12 in eq 9 then results in

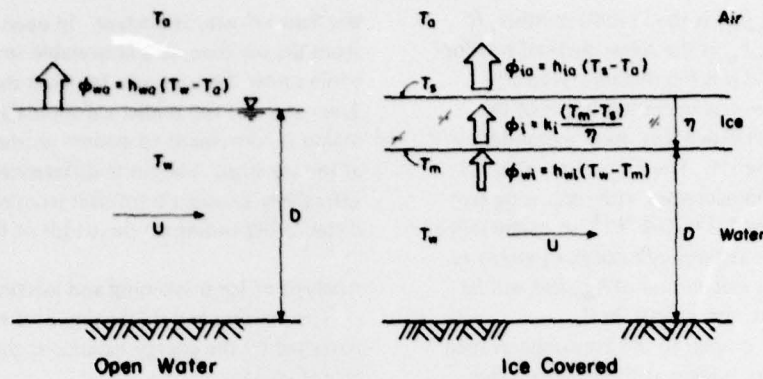


Figure 1. Definition sketch for heat transfer analysis.

$$\frac{T_m - T_a}{\frac{\eta}{k_i} + \frac{1}{h_{ia}}} - h_{wi}(T_w - T_m) = \rho_i \lambda \frac{d\eta}{dt} \quad (13)$$

which, if  $T_a$  and  $T_w$  are constant, may be readily integrated. They seldom are, however, so in the numerical analysis eq 13 will be integrated in step-wise fashion using

$$\Delta\eta = \frac{\Delta t}{\rho_i \lambda} \left[ \frac{T_m - T_a}{\frac{\eta}{k_i} + \frac{1}{h_{ia}}} - h_{wi}(T_w - T_m) \right] \quad (14)$$

and, of course, if the resulting value of  $\eta < 0$  after a time step, then  $\eta$  is set equal to 0 and open water exists.

#### Numerical simulation

Because the numerical simulation presented below considers the effect of variations in the initial effluent temperature with time, it is convenient to transform eq 2 to a Lagrangian coordinate system moving at the mean velocity of the flow. Thus, introducing  $Udt = dx$ , eq 2 becomes, for a parcel of water moving with the flow,

$$\frac{DT_w}{Dt} = \frac{\partial}{\partial z} \left( E_z \frac{\partial T_w}{\partial z} \right) - \frac{\phi}{\rho C_p D} \quad (15)$$

The explicit finite difference equation used in the simulation is

$$\begin{aligned} \frac{T_j^{i+1} - T_j^i}{\Delta t} &= \frac{(E_{j+1} + E_j) T_{j+1}^i - (E_{j+1} + 2E_j + E_{j-1}) T_j^i + (E_j + E_{j-1}) T_{j-1}^i}{2(\Delta z)^2} \\ &\quad - \frac{\phi}{\rho C_p D} \end{aligned}$$

$$- \frac{\phi}{\rho C_p D} \quad (16)$$

where the  $E$  values depend on whether or not the flow is ice-covered ( $E = E_{zi}$ ) or open ( $E = E_{zo}$ ) at the respective  $j$  grid points. The  $\phi$  term is also dependent on whether or not the flow is ice-covered and we choose the  $i, j$  location for the determination. Thus

$$\phi_{wi} = h_{wi} T_j^i \quad (\text{ice cover present}) \quad (17)$$

$$\phi_{wa} = h_{wa} (T_j^i - T_a) \quad (\text{open surface}). \quad (18)$$

Solving eq 16 explicitly for  $T_j^{i+1}$  results in

$$T_j^{i+1} = a T_{j-1}^i + b T_j^i + c T_{j+1}^i + d T_a \quad (19)$$

where the coefficients  $a, b, c$  and  $d$  are given by

$$a = \frac{\Delta t}{2(\Delta z)^2} (E_j + E_{j-1}) \quad (20)$$

$$b = 1 - \frac{\Delta t h_{wi}}{\rho C_p D} - \frac{\Delta t}{2(\Delta z)^2} (E_{j+1} + 2E_j + E_{j-1}) \quad (\text{ice cover}) \quad (21)$$

$$b = 1 - \frac{\Delta t h_{wa}}{\rho C_p D} - \frac{\Delta t}{2(\Delta z)^2} (E_{j+1} + 2E_j + E_{j-1}) \quad (\text{open surface}) \quad (22)$$

$$c = \frac{\Delta t}{2(\Delta z)^2} (E_{j+1} + E_j) \quad (23)$$



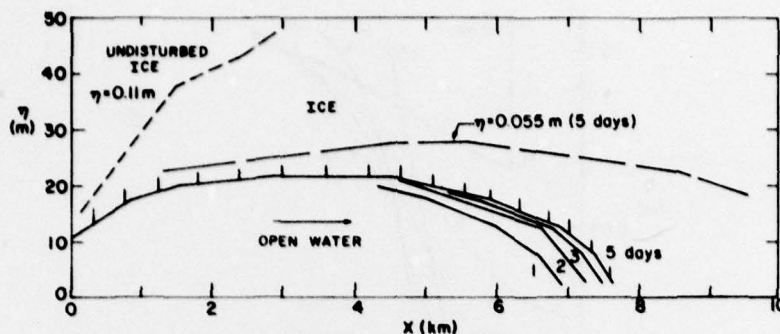


Figure 2. Results of example simulation for a smaller river under steady-state conditions.

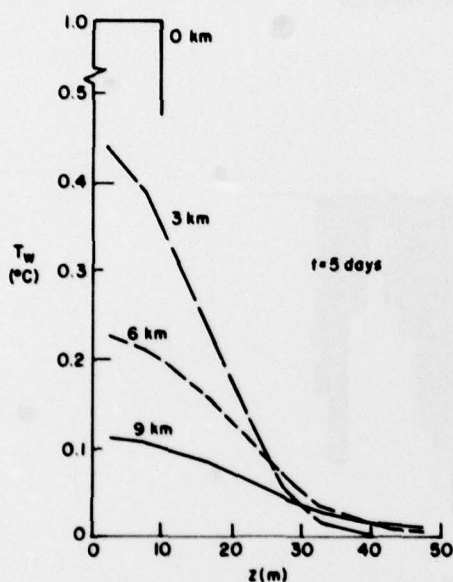


Figure 3. Lateral variation of water temperatures for example simulation after five days.

$$d = 0 \quad (\text{ice cover}) \quad (24)$$

$$d = \frac{\Delta t h_{wa}}{\rho C_p D} \quad (\text{open surface}). \quad (25)$$

Again, the value of the  $E$ 's depends on whether or not an ice cover or an open surface is present.

The above explicit scheme is stable for

$$\frac{2E \Delta t}{(\Delta z)^2} < 1 \quad (26)$$

and the appropriate value for  $E$  is, of course,  $E_0$ , since it is larger than  $E_1$  for the same total depth.

Initial conditions must also be specified. Since one seldom has detailed knowledge of ice thickness variations, the choice is practically between an initial uniform ice thickness or an initial open water condition. In either case, even for steady effluent temperatures and steady air temperatures, it takes some time for an equilibrium state to be reached. In the case of an initial zero thickness ice cover, this is due to the requirement of melting ice which has grown during the period before the thermal "front" reaches a downstream point. In the case of an initial uniform ice thickness, the time to reach equilibrium may be quite long because of the large thermal inertia of the ice cover. In the example simulations which follow the initial conditions have been taken as zero thickness and  $T_w = 0^\circ\text{C}$ . In application, the most reasonable approach would seem to be to run the simulation for several days prior to the period of interest unless, of course, the situation to be investigated is the one of abrupt release of a thermal effluent.

A complete listing of the computer program may be found in Appendix B.

#### Example simulations

In this section two example simulations are presented. The first example is that of a smaller river 50 m wide and 2 m deep with a mean velocity of  $0.5 \text{ m s}^{-1}$ . The thermal effluent is distributed over an initial width of 10 m at a temperature of  $1.0^\circ\text{C}$ , which corresponds to a heat load of 42 MW. An initial thickness of 0.0 m is assumed, and the air temperature is maintained constant at  $-5^\circ\text{C}$ . Other parameter values used are  $h_{wa} = 25 \text{ W m}^{-2}^\circ\text{C}^{-1}$  (corresponding approximately to a mean wind velocity of about  $4.5 \text{ m s}^{-1}$ ),  $k = 0.2$ ,  $U_a = 0.04 \text{ m s}^{-1}$ , and  $C = 0.023$ . A grid spacing of  $\Delta z = 10 \text{ m}$  and  $\Delta x = 300 \text{ m}$  ( $\Delta t = 600 \text{ s}$ ) was used.

In Figure 2 the ice edge location is shown for times of one, two, three, four and five days after initial thermal effluent release, and it is seen in the figure that

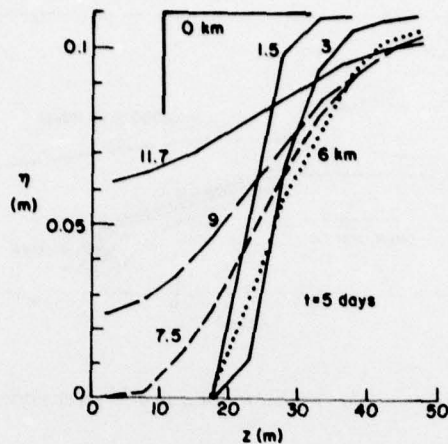


Figure 4. Lateral variation in ice thickness for example simulation after five days.

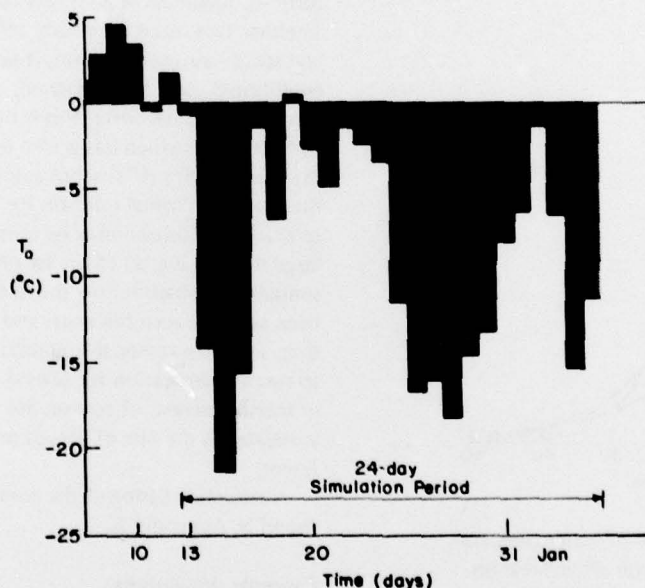


Figure 5. A typical winter daily air temperature variation used in the example simulation of a wide river.

equilibrium was achieved after about three days. Also shown in Figure 2 is the boundary of the region unaffected by the thermal effluent after five days (defined by an ice thickness differential of  $\Delta\eta < 0.001$  m for  $\eta = 0.11$  m) and the location at which the ice thickness was one-half its undisturbed value after five days. In Figures 3 and 4 are presented lateral profiles of water temperature  $T_w$  and ice thickness  $\eta$  respectively, at various distances downstream from the release point, in both cases for  $t = 5$  days. We also note that had the thermal effluent been fully mixed across the width of

the river at  $x = 0$  the ice edge would have been located in the vicinity of 1.5 km.

The second simulation is for a very wide river, in fact sufficiently wide that reflection from the far bank has no effect on the results both because of its distance and the decrease in  $T_w$  due to heat losses to the ice cover. The flow was characterized by a depth of 4 m and a mean velocity of  $0.5 \text{ m s}^{-1}$ , and a heat load of 540 MW was distributed over an initial width of 32 m with a corresponding water temperature of  $4.0^\circ\text{C}$ . This heat load was maintained constant while the air tem-

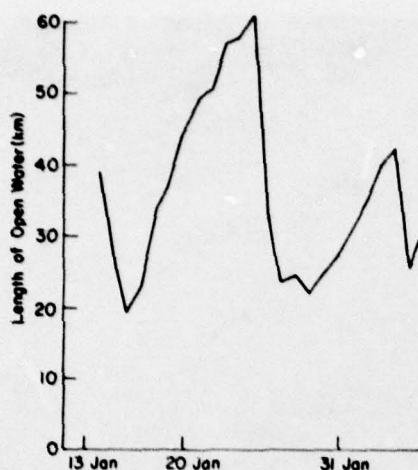


Figure 6. Variation of length of open water for the wide river example simulation.

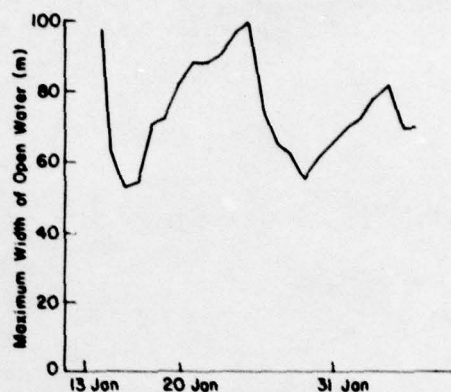


Figure 7. Variation of maximum width of open water for the wide river example simulation.

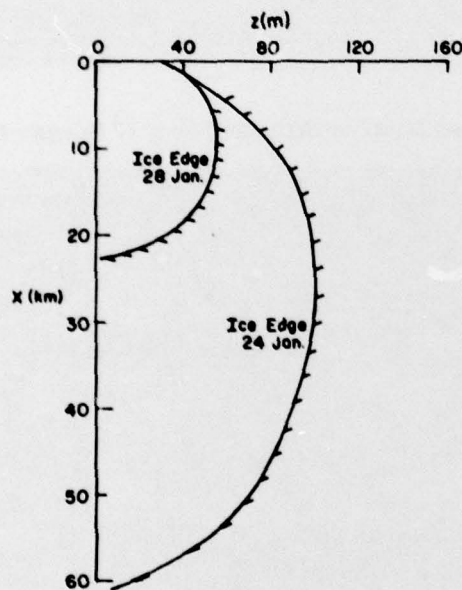


Figure 8. Maximum and minimum open water extents for the wide river example simulation.

perature varied as shown in Figure 5 (taken from an actual temperature record in the midwestern United States for 1972). An initial thickness of 0.0 m was used and other parameters were as follows:  $h_{wa} = 25 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1}$ ,  $k = 0.2$ ,  $U_a = 0.04 \text{ m s}^{-1}$ ,  $C = 0.03$ ,  $\Delta z = 16 \text{ m}$ ,  $\Delta x = 1350 \text{ m}$  (corresponding to  $\Delta t = 2700 \text{ s}$ ).

Figure 6 presents the resulting length of open water and Figure 7 the maximum width of open water, the location of which varied somewhat in the longitudinal direction (from about 6 km to 25 km). The extreme

cases of open water extent corresponding to 24 January (maximum extent) and 28 January (near minimum extent) are presented in Figure 8. The extreme foreshortening of the  $x$  scale relative to the  $z$  scale in Figure 8 (a ratio of 250:1) somewhat obscures how long and narrow are the open water extents. Finally we note that the example presented is also valid for a release at the centerline with a heat load of twice that used since the boundary condition at the bank ( $\partial T / \partial z = 0$ ) is the same by symmetry.

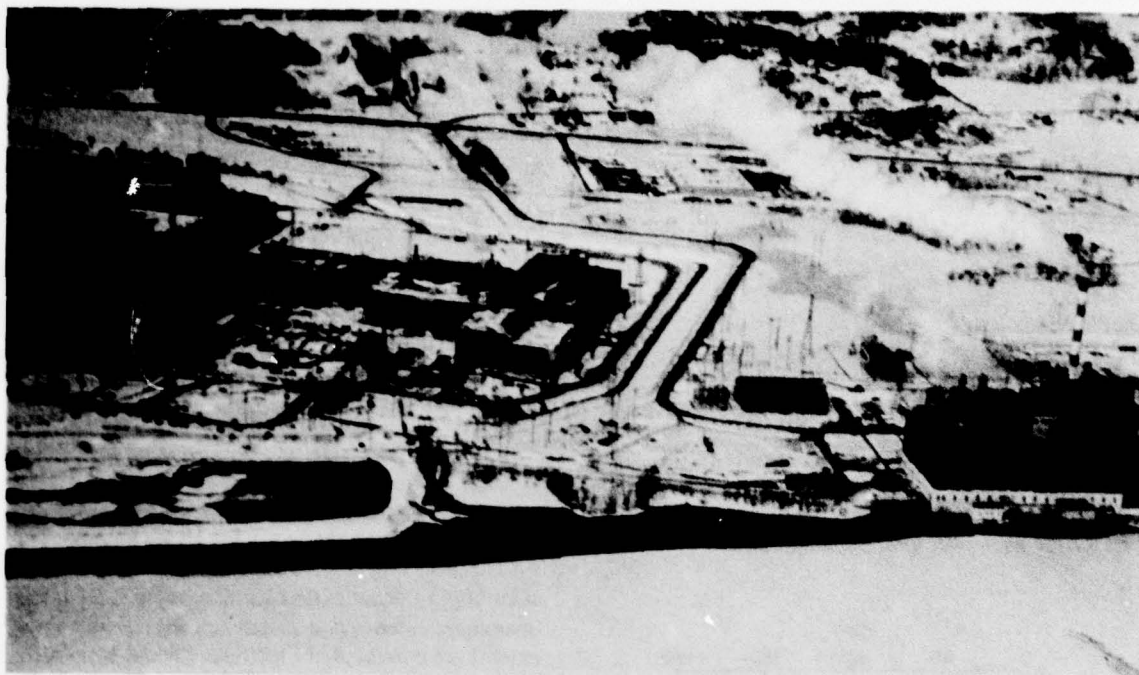
#### Field comparison

It is desirable to test the simulation against actual field observations. One very limited set of data was obtained of the open water that existed downstream of the Riverside Power Plant on the Mississippi River upstream of Bettendorf, Iowa. The data consisted of oblique angle aerial photography of the open water area and were obtained on 17 February 1979. Figure 9 shows a view of the power plant and near-field mixing zone. On the cover there is a view looking downstream from the vicinity of the plant. Figures 10 and 11 are similar views from points farther downstream. In Figure 11 an ice edge may be seen, although farther downstream there were numerous patches of open water in the plume area. Other data were obtained subsequent to the aerial photography; their sources are as follows:

**River hydrography:** 5 m depth fairly uniform along west side of river; obtained from hydrographic charts of the river.

**Mean velocity:**  $0.61 \text{ m s}^{-1}$ , obtained from a float measurement on 13 February 1979. River flow discharges were reasonably uniform over the period 13 to 17 February.





*Figure 9. Power plant and near field open water of the Riverside power plant near Bettendorf, Iowa, 17 February 1979.*



*Figure 10. View of open water several kilometers downstream from the Riverside power plant.*



*Figure 11. View downstream near end of open water extent. (Our apology for the poor quality of the photograph.)*

*Daily air temperatures:* average of daily maximum and daily minimum air temperatures recorded at the Quad-City Airport in Moline, Illinois, which is located about six miles from the river site.

*Initial width of mixing:* determined from aerial photography to be approximately 30 m.

*Effluent source temperature:* 2.97°C. This value was obtained using design temperature rise of water passing through the various plant units after correcting for plant capacity (figures

supplied by Iowa-Illinois Gas and Electric Company). The effluent water was then mixed over a width of 30 m of the river flow to obtain the 2.97°C. Considerable uncertainty exists in this value since only monthly plant operating figures were available. Further, the intake and outfall geometry are sufficiently complicated that a simple near-field mixing analysis was not feasible.

*Wind speed:* Not recorded. In lieu of site information,  $h_{wa}$  and  $h_{ia}$  were arbitrarily taken as

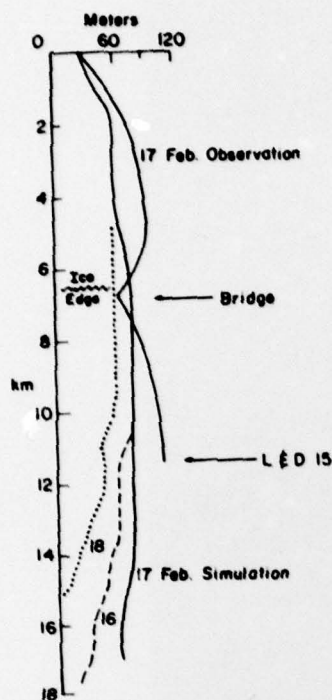


Figure 12. Comparison of numerical simulation results for 16, 17, 18 February and observed open water extent on 17 February.

$25 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1}$ ; this corresponds approximately to an average wind speed of  $4.5 \text{ m s}^{-1}$ .

Using the above information, three simulations were run on the computer using values for the mixing coefficient of  $k = 0.20, 0.40$  and  $0.80 \text{ m}^2 \text{ s}^{-1}$ . The last value provided the best agreement with observation of the three runs and the resulting open water extent is shown in Figure 12 for the days of 16, 17 and 18 February. Also shown is the observed open water extent on 17 February.

Some interpretation of Figure 12 is in order. Approximately 7 km downstream from the plant the ice edge appears to be pinched at the location of a suspension bridge over the river. This is perhaps explained by the presence of a pier located approximately at the ice edge location which may act to stabilize the cover. At a distance of 11 km Lock and Dam 15 severely changes the local flow geometry since all the river flow passes through roller gates on the opposite side of the river. Evidence of the thermal plume existed downstream of L & D 15 but no attempts were made to simulate the complex flow conditions resulting at the dam. The simulation was arbitrarily extended using the upstream depths. The "ice edge" shown in Figure

12 is that also depicted in Figure 11. However, extensive patches of open water existed in the thermal plume area downstream and the solid line shown in Figure 12 is the outline of this area. The accuracy of the simulation is difficult to evaluate from these limited data, both because of the uncertainty in the input parameters, and because of the rapid advance upstream of the ice edge, resulting from a four-day cold period. The simulation was performed using a transverse grid interval of 15 m, a total grid width of 150 m, and a time step of 409.8 s.

It is planned to conduct a more detailed field study at this site to include direct measurement of water temperature profiles, ice extent and meteorological variables. These data will allow a better determination of the quality of performance of the simulation.

#### Uncertainties and limitations

The uncertainties present in the analysis largely relate to the magnitude of the coefficients used.  $h_{wa}$  and  $h_{ia}$  (assumed equal in this analysis) can always be better approximated if more extensive meteorological information is available. The magnitude of  $C$  used to estimate  $h_{wi}$  should be verified by actual field measurements and one virtue of the present work is a means of estimating whether a particular field site is sufficiently well-mixed horizontally that measurements of temperatures in the streamwise direction will accurately yield a value of  $C$ . The ice edge location results from a melting (or freezing) analysis that assumes the cover is stable for very small values of  $\eta$ . This is probably reasonable in the lateral direction since at the ice edge the thickness changes rapidly with  $z$ . It is less certain at the downstream edge since the ice thickness tends to "feather" and be quite thin over fairly large  $x$  distances. Other limitations in location of the downstream extent are discussed in Part I.

The analysis has assumed a uniform distribution of velocity and constant depth in the  $z$  direction which is particularly convenient since time steps then correspond to distance steps. Holley et al. (1972) have discussed the effect of lateral variations in velocity and depth on the dispersion coefficient, and it can be significant. One improvement in the present analysis would be to incorporate such effects, perhaps even with a quasi-steady shift of the velocities towards the open areas as the cover is melted away. One procedure possible is that suggested by Sayre and Yeh (1973), who utilized the Manning relationship on a local basis to determine the transverse distribution of water discharge. Finally, secondary currents can result in considerable magnification of the effective value of  $k$  when the river meanders. Incorporation of these effects would



probably require an implicit algorithm rather than the conveniently simple explicit algorithm used herein.

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# APPENDIX A: UNSTEADY FULLY MIXED ICE SUPPRESSION

A \*06/05/78-09:24

```

1. C G ASHTON 24 APRIL 1978 UNSTEADY FULLY MIXED ICE SUPPRESSION
2. C THIS PROGRAM COMPUTES DOWNSTREAM EVOLUTION OF AN ICE COVER
3. C IN RESPONSE TO RELEASE OF A FULLY MIXED THERMAL EFFLUENT
4. C AND VARYING AIR TEMP; RIVER FLOW IS ASSUMED TO BE STEADY
5. C AL,D,U,ETA, TWOUT, H ARE LENGTH, DEPTH, MEAN VELOCITY,
6. C ICE THICKNESS, WATER TEMPERATURE, AND WIDTH FOR EACH SUBREACH
7. C DAT AND STW ARE DAILY AIR TEMPS AND EFFLUENT SOURCE TEMPS
8. DIMENSION SAL(60),SR(60),SD(60),DAT(60),STW(60)
9. DIMENSION AL(100),D(100),U(100),ETA(100),TWOUT(100),H(100)
10. DIMENSION CWAR(70),V(70)
11. C CWAR AND V ARE DAILY HEAT TRANSFER COEFF & WIND SPEED
12. READ 900,NT,NSR,ALTOT,DISCH
13. 900 FORMAT(2I10,2F10.0)
14. PRINT #16,NT,NSR,ALTOT,DISCH
15. 816 FORMAT(1H1,5X,'NT NSR ALTOT DISCH ',2I4,2F12.3)
16. C ALTOT IS LENGTH OF STUDY REACH (M), DISCH IS DISCHARGE (M3/S),
17. C NSR IS NUMBER OF SUBREACHES FOR WHICH VALUES OF WIDTH OF
18. C CHANNEL AND DEPTH ARE AVAILABLE
19. C NT IS NUMBER OF DAYS OF SIMULATION
20. READ 901,(SR(J),SD(J),SAL(J),J=1,NSR)
21. 901 FORMAT (3F10.7)
22. PRINT #17,(SR(J),SD(J),SAL(J),J=1,NSR)
23. 817 FORMAT(3X,' SR = ',F10.3,' SD = ',F10.3,' SAL = ',F10.3)
24. C SB AND SD ARE WIDTH AND DEPTH AT SUCCESSIVE X DISTANCES, SAL
25. C SUBDIVIDE TOTAL REACH INTO SUBREACHES WITH DELT TRAVEL TIME
26. DELT = 3600.
27. C CALCULATE NL AND TOTAL TRAVEL TIME
28. TRAV = 0.0
29. DO 10 J=1,NSR
30. 10 TRAV=TRAV+SR(J)*SD(J)*SAL(J)/DISCH
31. NL=TRAV/DELT
32. TRAV=TRAV/3600.
33. PRINT #10,TRAV
34. 810 FORMAT(10X,' TOTAL TIME OF TRAVEL = ',F10.2,' HOURS')
35. C CALCULATE NEW SUBREACH LENGTHS WIT DELT TIME OF TRAVEL
36. SUMSAL=SAL(1)
37. SUMAL=0.0
38. I=1
39. DO 15 J=1,NL
40. U(J)=DISCH/(SR(I)*SD(I))
41. AL(J)=DELT*U(J)
42. D(J)=SD(I)
43. SUMAL=SUMAL+AL(J)
44. DELAL=SUMAL-SUMSAL
45. IF (DELAL)15,15,12
46. 12 I=I+1
47. SUMSAL=SUMSAL+SAL(I)
48. 15 CONTINUE
49. C ROUTINE ONLY GOOD IF ALL SAL LONGER THAN DELT*U(J)
50. PRINT #11
51. 811 FORMAT(5X,' AL(J)',3X,'U(J)')
52. PRINT #13
53. 813 FORMAT(3X,' METERS',3X,'M/S')
54. PRINT #12,(AL(J),U(J),J=1,NL)
55. 812 FORMAT(2F10.2)
56. C READ AIR TEMPERATURES AND SOURCE TEMPS FOR NT DAYS
57. READ 902,(DAT(I),STW(I),V(I),I=1,NT)
58. 902 FORMAT(3F10.0)
59. PRINT #15
60. 815 FORMAT(3X,' T',3X,' AIR T',3X,' WATER T)
61. PRINT #14,(I,DAT(I),STW(I),I=1,NT)
62. 814 FORMAT(15,2F10.4)
63. C NOW INITIALIZE PROPERTIES AND COEFFICIENTS
64. CP=4215.
65. AKI=2.24

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66.      ALAM=5.34E5
67.      AMU=1.79E-5
68.      RHOI=916.
69.      RHOW=1000.
70.      CUI=1622.
71.      CWA=25.
72.      C      SET ICE COVER THICKNESS AND WATER TEMPERATURE AT ZERO
73.      DO 25 I=1,NL
74.      ETA(I)=0.0
75.      25      TWOUT(I)=0.0
76.      NTD=86400./DELT
77.      DO 30 I=1,NT
78.      30      CWAR(I)=4.5+3.*V(I)
79.      50      DO 400 IT=1,NT
80.      CWA=CWAR(IT)
81.      100     DO 390 IOT=1,NTD
82.      TWOUT(I)=STW(IT)
83.      C      ESTABLISHES WATER TEMP FOR INLET 1ST SURFACH
84.      DO 380 J=1,NL
85.      IF(ETA(J))250,250,300
86.      C      NO ICE COVER
87.      250     QW=CWA*(TWOUT(J)-DAT(IT))
88.      DELTW=-QW*DELT/(RHOW*CP*D(J))
89.      TWOUT(J)=TWOUT(J)+DELTW
90.      C      OUTLET TEMPERATURE FOR SURFACH AT END OF IOT TIME STEP
91.      IF(TWOUT(J))260,270,270
92.      260     TWOUT(J)=0.0
93.      ETA(J)=DELT*CWA*(-DAT(IT))/(RHOW*ALAM)
94.      270     GO TO 380
95.      C      ICE COVER PRESENT
96.      300     QW=CUI*((U(J)**0.8)/(D(J)**0.2))*TWOUT(J)
97.      DELTW=-QW*DELT/(RHOW*CP*D(J))
98.      TWOUT(J)=TWOUT(J)+DELTW
99.      IF(DAT(IT))306,305,305
100.      305     QI=0.0
101.      GO TO 307
102.      306     QI=-DAT(IT)/((ETA(J)/AKI)+(1./CWA))
103.      DETA=(DELT/(RHOI*ALAM))*(-QW+QI)
104.      307     CONTINUE
105.      ETA(J)=ETA(J)+DETA
106.      IF(ETA(J))320,380,380
107.      320     ETA(J)=0.0
108.      380     CONTINUE
109.      C      PRINT 801,(J,ETA(J),TWOUT(J),J=1,NL,20)
110.      DO 385 J=NL,2,-1
111.      385     TWOUT(J)=TWOUT(J-1)
112.      390     CONTINUE
113.      ALO=0.0
114.      DO 395 J=1,NL
115.      IF(ETA(J))391,391,395
116.      391     ALO=ALO+AL(J)
117.      395     CONTINUE
118.      PRINT 802,IT
119.      PRINT 851
120.      851     FORMAT(3X,'ICE THICKNESSES')
121.      PRINT 801,(ETA(J),J=1,NL)
122.      PRINT 803,ALO
123.      PRINT 852
124.      852     FORMAT(3X,'WATER TEMPERATURES')
125.      PRINT 801,(TWOUT(J),J=1,NL)
126.      400     CONTINUE
127.      803     FORMAT(10X,' ALO = ',F12.3,' METERS')
128.      802     FORMAT(1H0,'END OF DAY',I5)
129.      801     FORMAT(10F7.3)
130.      END

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# APPENDIX B: UNSTEADY LATERAL MIXING ICE SUPPRESSION

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1A *05/23/78=08:06
1. C G ASHTON 22 MAY 1978
2. C UNSTEADY LATERAL MIXING ICE SUPPRESSION
3. DIMENSION AI(100),ETA(100,20),TW(100,20),DAT(60),STW(60)
4. READ 401,NT,NSR,NSW
5. READ 402,DELT
6. READ 403,(DAT(IT),IT=1,NT)
7. READ 404,(STW(IT),IT=1,NT)
8. READ 405,UM,DM,RW
9. READ 406,CW
10. 401 FORMAT (3I10)
11. 402 FORMAT (F10.0)
12. 403 FORMAT(F10.0)
13. 404 FORMAT (F10.0)
14. 405 FORMAT(3F10.4)
15. 406 FORMAT(I10)
16. C NOW INITIALIZE WATER TEMPERATURE AND ICE THICKNESS
17. DO 50 I=1,NSR
18. DO 40 J=1,NSW
19. ETA(I,J)=0.0
20. TW(I,J)=0.0
21. 40 CONTINUE
22. 50 CONTINUE
23. C PRINT OUT INPUT DATA
24. PRINT 501,NT,NSR,NSW
25. 501 FORMAT(3X,' NT NSR NSW = ',3I10)
26. PRINT 502,DELT
27. 502 FORMAT(3X,' DELT = ',F12.1,' SECONDS')
28. PRINT 503
29. 503 FORMAT(3X,' DAY TAC( DEG C ) TW( O')
30. PRINT 504,(IT,DAT(IT),STW(IT),IT=1,NT)
31. 504 FORMAT(3X,I4,2F10.2)
32. PRINT 505
33. 505 FORMAT(3X,' INITIAL ICE THICKNESSES')
34. PRINT 506,((ETA(I,J),J=1,NSW),I=1,5 )
35. 506 FORMAT(20F5.3)
36. PRINT 507
37. 507 FORMAT(100,' INITIAL WATER TEMPERATURES')
38. PRINT 508,((TW(I,J),J=1,NSW),I=1,5 )
39. 508 FORMAT(20F5.2)
40. C NOW ESTABLISH LENGTHS WITH TIME OF TRAVEL EQUAL TO DELT
41. ALSR=UM*DELT
42. PRINT 509,UM
43. 509 FORMAT(100,3X,'MEAN VELOCITY = ',F10.3,' M PER SEC')
44. PRINT 510,DM
45. 510 FORMAT(3X,'MEAN DEPTH = ',F10.3,' METERS')
46. PRINT 511,ALSR
47. 511 FORMAT(3X,' SURFEACH LENGTH = ',F12.1,' METERS')
48. DO 60 I=1,NSR
49. AI(I)=ALSR
50. 60 CONTINUE
51. PRINT 512,RW
52. 512 FORMAT(3X,'TOTAL WIDTH = ',F12.1,' METERS')
53. DELR=RW/NSW
54. C NOW DETERMINE NUMMR OF TIME STEPS PER DAY NTD
55. NTD=86400./DELT
56. C NOW INITIALIZE PROPERTIES AND COEFFICIENTS
57. CP=4215.
58. AKI=2.24
59. ALAM=3.34E5
60. RHOI=916.
61. RHOW=1000.
62. CWI=1622.
63. CWA=25.
64. HWA=CWA
65. HWI=CWI*(UM**0.8)/(DM**0.2)

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66.      F=0.05
67.      C      F IS ARBITRARILY CHOSEN FOR EXAMPLE CASE
68.      USTAR=H*SQRT(F/P.)
69.      EK=0.2
70.      C      CALCULATE STABILITY PARAMETER
71.      C      STABLE = 2.*EK*USTAR*DM*DEL/(DELH*DELH)<1
72.      STABLE=2.*EK*USTAR*DM*DEL/(DELH*DELH)
73.      IF(STABLE-1.)70,71,71
74.      70      PRINT 515,STABLE
75.      515      FORMAT(3X,'STABILITY PARAMETER = ',F6.3,' STABLE')
76.      GO TO 72
77.      71      PRINT 516,STABLE
78.      516      FORMAT(3X,'STABILITY PARAMETER = ',F6.3,' UNSTABLE')
79.      72      CONTINUE
80.      PRINT 517,EK
81.      517      FORMAT(3X,'EK = ',F6.3,' M2 PER SEC')
82.      PRINT 518,USTAR
83.      518      FORMAT(3X,'USTAR = ',F7.4,' M PER SEC')
84.      PRINT 521,DELH
85.      521      FORMAT(3X,'DELH = ',F7.1,' METERS')
86.      EO=EK*USTAR*DM
87.      EI=EO/2.
88.      RCD=RHOW*CP*DM
89.      DT=DEL/(2.*DELH*DELH)
90.      C*****
91.      DO 700 IT=1,NT
92.      DO 450 ID=1,NTD
93.      C      INITIALIZE UPSTREAM WATER TEMPERATURE VARIATION
94.      DO 130 J=1,NIW
95.      TW(1,J)=STW(IT)
96.      130      CONTINUE
97.      DO 650 I=NSR,2,-1
98.      C      CALCULATE J=1 NODE
99.      IF(ETA(IT))133,133,134
100.     133      EJP1=EO
101.      GO TO 135
102.     134      EJP1=ET
103.     135      CONTINUE
104.      IF(ETA(IT))136,136,137
105.     136      EJ=EO
106.      R=1.-(DEL*HWA/RCD)-DT*(EJP1+  EJ)
107.      D=DEL*HWA/RCD
108.      GO TO 138
109.     137      EJ=EI
110.      R=1.-DEL*HWT/RCD-DT*(EJP1+  EJ)
111.      D=0.
112.     138      CONTINUE
113.      C=DT*(EJP1+EJ)
114.      TW(IT,1)=R*TW(IT-1,1)+C*TW(IT-1,2)+D*DAT(IT)
115.      C      CALCULATE J=NSW NODE
116.      IF(ETA(IT,NSW))151,151,152
117.     151      EJM1=EO
118.      GO TO 153
119.     152      EJM1=ET
120.     153      CONTINUE
121.      IF(ETA(IT,NSW))154,154,155
122.     154      EJ=EO
123.      R=1.-DEL*HWA/RCD-DT*(EJM1+  EJ)
124.      D=DEL*HWA/RCD
125.      GO TO 156
126.     155      EJ=EI
127.      R=1.-DEL*HWT/RCD-DT*(EJM1+  EJ)
128.      D=0.
129.     156      CONTINUE
130.      A=DT*(EJ+EJM1)
131.      TW(IT,NSW)=A*TW(IT-1,NSW-1)+R*TW(IT-1,NSW)+D*DAT(IT)
132.      C      CALCULATE INTERMEDIATE NODE POINTS

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133.      DO 600 J=2, NSW-1
134.      IF (ETA(I,J-1)) 165, 165, 166
135.      165      EJM1=EO
136.      GO TO 167
137.      166      EJM1=ET
138.      167      CONTINUE
139.      IF (ETA(I,J)) 168, 168, 169
140.      168      EJM=EO
141.      GO TO 170
142.      169      EJM=ET
143.      170      CONTINUE
144.      IF (ETA(I,J+1)) 171, 171, 172
145.      171      EJP1=EO
146.      H=1.-DELTA*HWA/RCD-DT*(EJP1+2.*EJ+EJM1)
147.      D=DELTA*HWA/RCD
148.      GO TO 173
149.      172      EJP1=ET
150.      H=1.-DELTA*HWT/RCD-DT*(EJP1+2.*EJ+EJM1)
151.      D=0.
152.      173      CONTINUE
153.      A=DT*(EJ+EJM1)
154.      C=DELTA*(EJ+EJP1)
155.      TW(I,J)=A*TW(I-1,J-1)+B*TW(I-1,J)+C*TW(I-1,J+1)+D*DAT(IT)
156.      600      CONTINUE
157.      630      CONTINUE
158.      C      NOW CALCULATE ICE THICKNESSES AT J NODES
159.      DO 206 I=1, NSR
160.      DO 205 J=1, NSW
161.      IF (DAT(IT)) 200, 200, 201
162.      200      Q1=-DAT(IT)/(CETA(I,J)/AKT)+1./HWA)
163.      GO TO 202
164.      201      Q1=0.
165.      202      CONTINUE
166.      QH=Q1*(CM**0.8)+TW(I,J)/(CM**0.2)
167.      DELETA=(Q1-QH)*DELTA/(QH01*ALPHA)
168.      ETA(I,J)=ETA(I,J)+DELETA
169.      IF (ETA(I,J)) 203, 203, 204
170.      203      ETA(I,J)=0.0
171.      204      CONTINUE
172.      205      CONTINUE
173.      206      CONTINUE
174.      650      CONTINUE
175.      C      PRINT OUT DAILY RESULTS
176.      PRINT 519, IT
177.      PRINT 520, ((TW(I,J), J=1, NSW), I=1, NSR)
178.      519      FORMAT (1H1, ' TW DAILY END OF DAY ', 15)
179.      520      FORMAT (10X, 10F6.3)
180.      PRINT 529, IT
181.      PRINT 530, ((ETA(I,J), J=1, NSW), I=1, NSR)
182.      529      FORMAT (1H1, ' ICE THICKNESSES END OF DAY ', 15)
183.      530      FORMAT (10X, 10F6.3)
184.      700      CONTINUE
185.      END

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